Java Runes

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Abstract

At the 2013 Bridges Conference I presented a scheme to create patterns of arcs within circles that represent functions. This idea has been extended to circles in any modulo with beautiful results and surprising number patterns that inspire deep mathematical thinking. I discuss a web app I created for touch devices that allows play and exploration with these designs.

Introduction

At Bridges 2013 I presented a paper and activity called "Math Runes." 10 points are arranged on a circle and the points are connected with lines or arcs to show the result of an operation, multiplication by 2 for example, modulo 10. The resulting shapes are delightful and encourage further mathematical and artistic exploration.

Any operation can be used, but multiplication is especially interesting. Figure 1 shows the runes for "× 0" to "× 9" made with arcs. (When constructed with straight lines, the designs resemble Viking runes, thus the name "math runes"). Pupils in my math art classes made surprising conjectures and connections between deep ideas in mathematics [1], and were eager to try making math runes with different function and in different moduli.

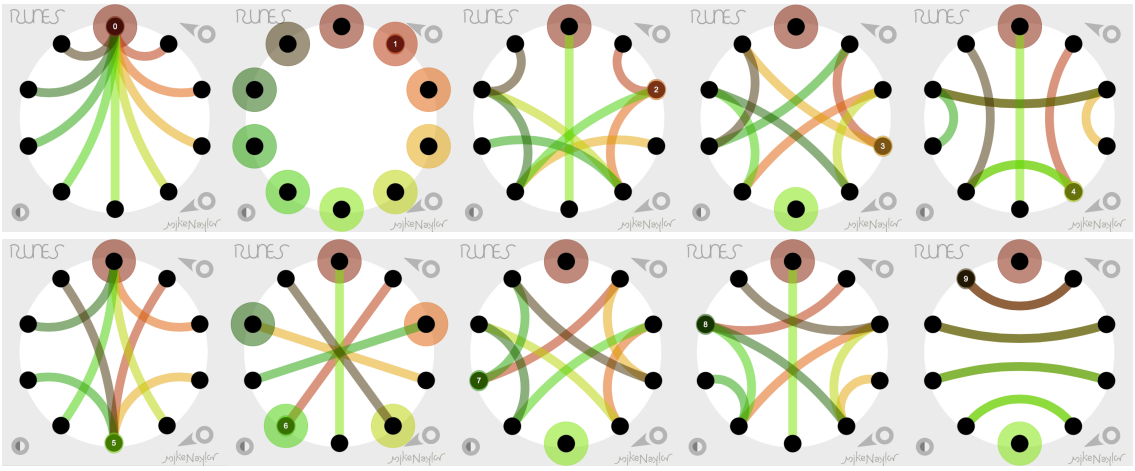

Figure 1: *Multiplication Runes from "× 0" to "× 9" in mod 10*

I have created a web app in Processing to create these runes in different moduli with different color schemes and animate changes from one multiplier to the another [2]. It is optimized for use with tablets or mobile touch devices like smartphones.

How it works

Each rune is a picture of multiplication. Every dot is a number with 0 at the top and then numbered clockwise around the circle, like a clock. The dots are numbered in the picture to the right, but in the app only one dot is numbered: the multiplier dot.

In this example shown in Figure 2, the multiplier is 2, so this "math rune" shows what happens to every number when multiplied by 2. There is an arc from 1 to 2, and 2 to 4, and 3 to 6, and so on.

5 on the bottom goes to 10, which is the same as 0 because the circle wraps around and starts over. We call this "modulo 10". Clocks are mod 12.

These are the multiplication runes, and there are fun pencil and paper activities to do with them. A link on the web app page has activities, articles and copy originals which can be useful for teachers.

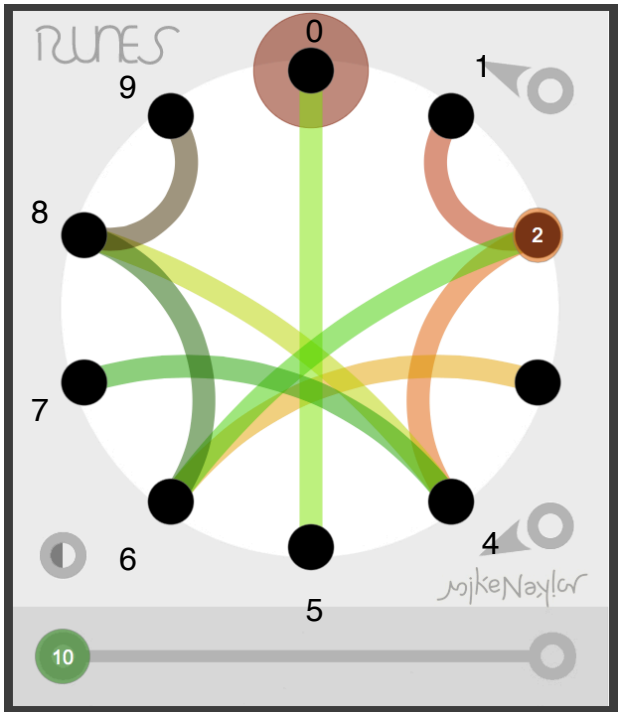

Figure 2: *The × 2 rune mod 10*

Other moduli

By adjusting the slider on the bottom of the screen, the number of dots can change from 10 to 360.

Figure 3 shows how the × 2 runes look like in mod 10, 20, 40 and 80. The unusual shape evolves into a heart form!

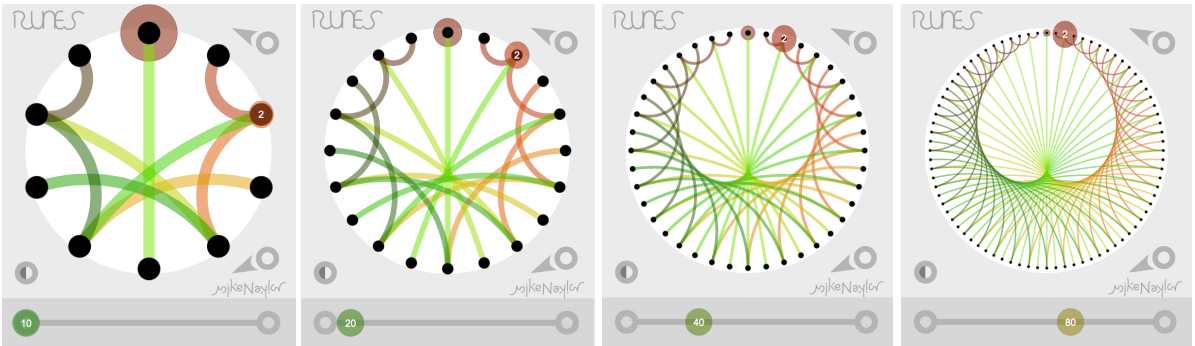

Figure 3: *Varieties of × 2 runes*

Animation: The multiplier can be touched and dragged directly, or it will animate from one multiplier to another automatically. The animation can be turned on and off by tapping in the middle of the screen. There are also buttons for changing the direction of the animation.

There are many surprising shapes and patterns that appear in different areas of the circle. Figure 4 shows × 2, × 3 and × 4 runes in mod 360. Each step up in the multiplier adds a leaf to the flower. *How long does the pattern continue? Where else do we find leaves and what is the connection to the numbers?*

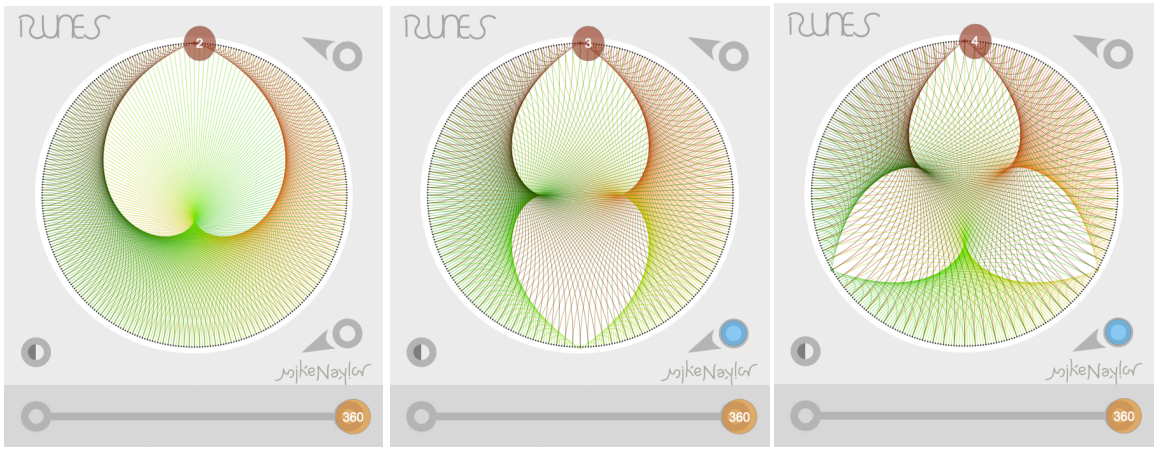

Figure 4: *× 2, × 3, and × 4 runes*

Color modes

A button above the slider and to the left changes between color schemes, shown in Figure 5. The original colors each arc according to the position of its dot of origin: (rgb(x,y,0)). The inverted color scheme has a neon effect that resembles string art designs done over a black background. A black and white scheme with extra fine lines shows good detail when using many points.

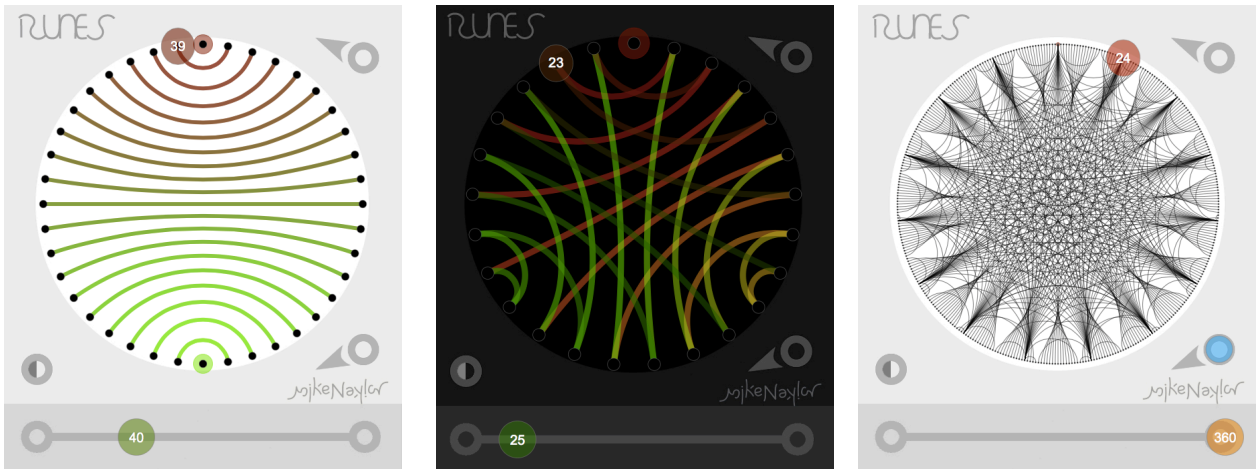

Figure 5: *Different color modes*

Another color scheme displays a circle in 12 colored sections, like a clock. This makes it easy to find fractions like halves, thirds, fourths and sixths, which can be interesting places to hunt for patterns. The fifths are marked with black stripes.

The following pictures give some ideas for what kind of surprises can be found. Figure 6 shows × 30, × 20 and × 12 runes in mod 60, which can be seen with the fraction ring to be 1/2, 1/3 and 1/5. Figure 7 shows the runes that come "just before" those three runes: × 29, × 19, and × 11.

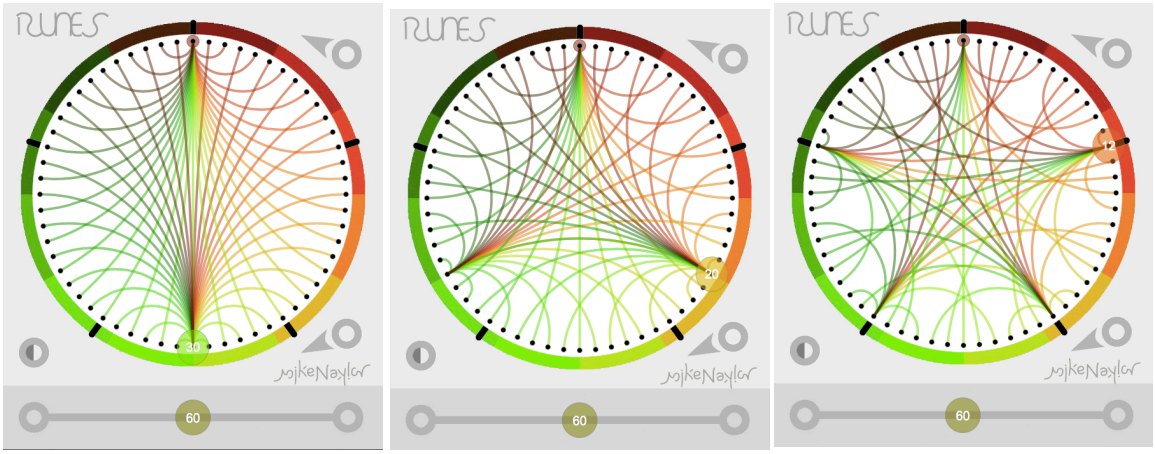

Figure 6: *× 30, × 20 and × 12 runes in mod 60*

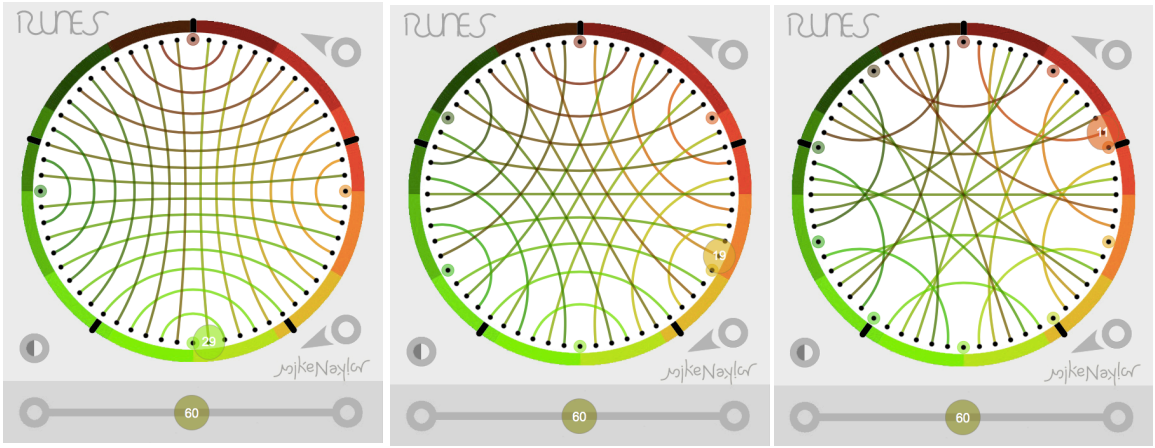

Figure 7: *One fewer... × 29, × 19 and × 11 runes in mod 60*

How do the fractions compare with the numbers? What kinds of symmetries are in the designs and what is the connection to the numbers? There are many ideas connecting fractions, divisors, multiples, least common multiples and greatest common divisors.

References

[1] Naylor, M. "Math Runes." 2013 Bridges Conference Proceedings, pp. 191-198, Enschede, Netherlands, July 2013.

[2] Naylor, M. "Math Runes web app." http://mike-naylor.com/runes, July 2014.